

Check of the Bootstrap Conditions for the Gluon Reggeization

Alessandro Papa

*Dipartimento di Fisica, Università della Calabria & INFN - Cosenza
I-87036 Arcavacata di Rende (Cosenza), Italy*

Abstract

The property of gluon Reggeization plays an essential role in the derivation of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation for the cross sections at high energy \sqrt{s} in perturbative QCD. This property has been proved to all orders of perturbation theory in the leading logarithmic approximation and it is assumed to be valid also in the next-to-leading logarithmic approximation, where it has been checked only to the first three orders of perturbation theory. From s -channel unitarity, however, very stringent “bootstrap” conditions can be derived which, if fulfilled, leave no doubts that gluon Reggeization holds.

The BFKL equation [1] is very important for the theory of Regge processes at high energy \sqrt{s} in perturbative QCD, such as the deep inelastic ep scattering in the region of small values of the Bjorken variable x , presently investigated at HERA. It was derived more than twenty years ago in the leading logarithmic approximation (LLA) [1], which means summation of all the terms of the type $(\alpha_s \ln s)^n$. Recently, the radiative corrections to the equation have been calculated (see Ref. [1] for a review) and the explicit form of the kernel of the equation in the next-to-leading approximation (NLA) for the case of forward scattering became known [1].

The key role in the derivation of the BFKL equation is played by the gluon Reggeization. “Reggeization” of a given elementary particle usually means that the amplitudes of the scattering processes with exchange of the quantum numbers of that particle in the t -channel go like $s^{j(t)}$ in the Regge limit $s \gg |t|$. The function $j(t)$ is called “Regge trajectory” of the given particle and takes the value of the spin of that particle when t is equal to its squared mass. It is well-known that in QED the electron Reggeizes in perturbation theory [2], while the photon remains elementary [3], while in QCD the gluon Reggeizes [4, 5] as well as the quark [6]. In perturbative QCD, the notion of gluon Reggeization is used in a stronger sense. It means not only that a Reggeon exists with the quantum numbers of the gluon, negative signature and with a trajectory $j(t) = 1 + \omega(t)$ passing through 1 at $t = 0$, but also that this Reggeon gives the leading contribution in each order of perturbation theory to the amplitudes of the processes with large s and fixed (i.e. not growing with s) squared momentum transfer t .

The simplest realization of the gluon Reggeization is in the elastic process $A + B \longrightarrow A' + B'$ with exchange of gluon quantum numbers in the t -channel (see Fig. 1), whose amplitude in the Regge limit takes the form

$$\left(\mathcal{A}_s^-\right)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)} \right] \Gamma_{B'B}^c. \quad (1)$$

Here c is a color index and $\Gamma_{P'P}^c$ are the particle-particle-Reggeon (PPR) vertices, not depending on s . They can be written in the form $\Gamma_{P'P}^c = g\langle P'|T^c|P\rangle\Gamma_{P'P}$, where g is

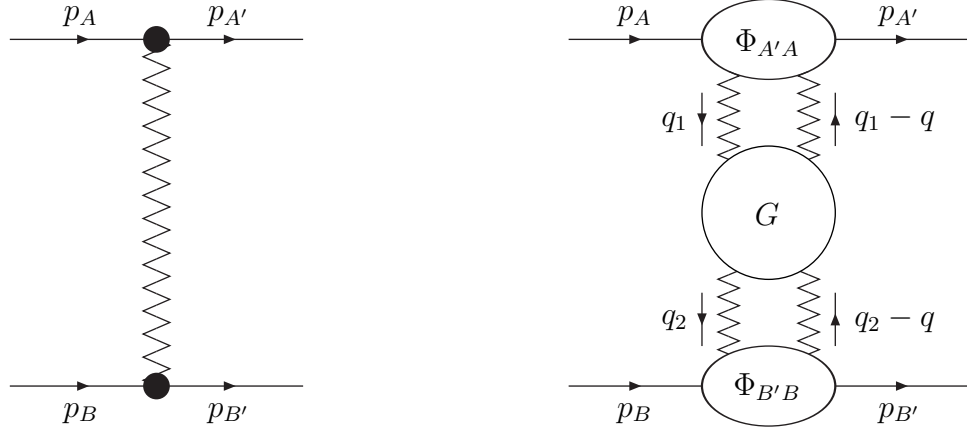


Figure 1: (Left) Diagrammatical representation of $(\mathcal{A}_8^-)_{AB}^{A'B'}$. The zig-zag line is the Reggeized gluon, the black blobs the PPR effective vertices. (Right) Diagrammatical representation of $(\mathcal{A}_R)_{AB}^{A'B'}$ from s -channel unitarity. The ovals are the impact factors of the particles A and B , the circle is the Green function for the Reggeon-Reggeon scattering.

the QCD coupling constant and T^c are the color group generators in the fundamental (adjoint) representation for quarks (gluons)¹. In the LLA this form of the amplitude has been rigorously proved [7]: $\Gamma_{P'P}^{(0)}$ is given simply by $\delta_{\lambda_{P'}, \lambda_P}$, where λ_P is the helicity of the particle P , and the Reggeized gluon trajectory is needed with 1-loop accuracy [1],

$$\omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2} k_\perp}{k_\perp^2 (q - k)_\perp^2}. \quad (2)$$

Here $D = 4 + 2\epsilon$ has been introduced in order to regularize the infrared divergences and the integration is performed in the space transverse to the momenta of the initial colliding particles. In the NLA (resummation of the terms $\alpha_s^{n+1}(\ln s)^n$), the form (1) has been checked in the first three orders of perturbation theory [1] and is only assumed to be valid to all orders. In this approximation, the NLA contribution to the PPR vertices is needed, which takes the form $\Gamma_{P'P}^{(1)} = \delta_{\lambda_{P'}, \lambda_P} \Gamma_{PP}^{(+)} + \delta_{\lambda_{P'}, -\lambda_P} \Gamma_{PP}^{(-)}$, where a helicity non-conserving term appeared, and the Reggeized gluon trajectory enters with 2-loop accuracy.

On the other hand, the amplitude for the elastic scattering process $A + B \rightarrow A' + B'$ (for arbitrary color representation in the t -channel) can be determined from s -channel unitarity, by expressing its imaginary part in terms of the inelastic amplitudes $A + B \rightarrow \tilde{A} + \tilde{B} + \{n\}$ and $A' + B' \rightarrow \tilde{A} + \tilde{B} + \{n\}$, and then by reconstructing the full amplitude by use of the dispersion relations. Of course, the additional particles $\{n\}$ to be considered in the intermediate states and their kinematical configurations depend on the adopted approximation. It turns out (see for instance [1, 8]) that this amplitude can be written as (see Fig. 1)

$$\begin{aligned} (\mathcal{A}_R)_{AB}^{A'B'} &= \frac{i s}{(2\pi)^{D-1}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2} q_2}{\vec{q}_2^2 \vec{q}_2'^2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{\sin(\pi\omega)} \left[\left(\frac{-s}{s_0} \right)^\omega - \tau \left(\frac{s}{s_0} \right)^\omega \right] \\ &\times \sum_{\mathcal{R}, \nu} \Phi_{A'A}^{(\mathcal{R}, \nu)}(\vec{q}_1, \vec{q}; s_0) G_\omega^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \Phi_{B'B}^{(\mathcal{R}, \nu)}(-\vec{q}_2, -\vec{q}; s_0), \end{aligned} \quad (3)$$

where \mathcal{A}_R stands for the scattering amplitude with the representation \mathcal{R} of the color group in the t -channel, the index ν enumerates the states in the irreducible representation \mathcal{R} and

¹Throughout this paper only the case of quarks and gluons as colliding particles is considered.

$G_{\omega}^{(\mathcal{R})}$ is the Mellin transform of the Green function for the Reggeon-Reggeon scattering [8]. The signature τ is positive (negative) for symmetric (antisymmetric) representation \mathcal{R} . The parameter s_0 is an arbitrary energy scale introduced in order to define the partial wave expansion of the scattering amplitudes through the Mellin transform. The dependence on this parameter disappears in the full expressions for the amplitudes. $\Phi_{P'P}^{(\mathcal{R},\nu)}$ are the so-called impact factors, defined by

$$\begin{aligned} \Phi_{P'P}^{(\mathcal{R},\nu)}(\vec{q}_R, \vec{q}; s_0) &= \int \frac{ds_{PR}}{2\pi s} \text{Im} \mathcal{A}_{P'P}^{(\mathcal{R},\nu)}(p_P, q_R; \vec{q}; s_0) \theta(s_{\Lambda} - s_{PR}) \\ &- \frac{1}{2} \int \frac{d^{D-2}q'}{\vec{q}'^2 (\vec{q}' - \vec{q})^2} \Phi_{P'P}^{(\mathcal{R},\nu)}(\vec{q}', \vec{q}) \mathcal{K}_r^{(\mathcal{R})B}(\vec{q}', \vec{q}_R) \ln \left(\frac{s_{\Lambda}^2}{(\vec{q}' - \vec{q}_R) s_0} \right), \end{aligned} \quad (4)$$

where $\mathcal{A}_{P'P}$ is the particle-Reggeon scattering amplitude. This definition is valid both in the LLA and in the NLA. In the former case, the second term in the R.H.S. of the above equation as well as the θ function in the first term are not active. In the NLA, instead, the second term behaves as a counterterm for the large s_{PR} contribution to the first integral, already taken into account in the LLA. The parameter s_{Λ} disappears in the final expression for the NLA impact factors.

The Green function obeys the generalized BFKL equation

$$\omega G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) = \vec{q}_1^2 \vec{q}_1'^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_r}{\vec{q}_r'^2 \vec{q}_r^2} \mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_r; \vec{q}) G_{\omega}^{(\mathcal{R})}(\vec{q}_r, \vec{q}_2; \vec{q}), \quad (5)$$

where we have introduced the notation $q'_i \equiv q_i - q$. The kernel $\mathcal{K}^{(\mathcal{R})}$ consists of two parts, a “virtual” part, related with the Reggeized gluon trajectory, and a “real” part, related to particle production:

$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \left[\omega(-\vec{q}_1^2) + \omega(-\vec{q}_1'^2) \right] \vec{q}_1^2 \vec{q}_1'^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) \quad (6)$$

The “real” part of the kernel is defined by

$$\begin{aligned} \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) &= \int \frac{ds_{RR}}{(2\pi)^D} \text{Im} \mathcal{A}_{RR}^{(\mathcal{R})}(q_1, q_2; \vec{q}) \theta(s_{\Lambda} - s_{RR}) \\ &- \frac{1}{2} \int \frac{d^{D-2}q'}{\vec{q}'^2 (\vec{q}' - \vec{q})^2} \mathcal{K}_r^{(\mathcal{R})B}(\vec{q}_1, \vec{q}'; \vec{q}) \mathcal{K}_r^{(\mathcal{R})B}(\vec{q}', \vec{q}_2; \vec{q}) \ln \left(\frac{s_{\Lambda}^2}{(\vec{q}' - \vec{q}_1)^2 (\vec{q}' - \vec{q}_2)^2} \right), \end{aligned} \quad (7)$$

where $\mathcal{A}_{RR}^{(\mathcal{R})}$ is the Reggeon-Reggeon scattering amplitude. Here the same arguments concerning s_{Λ} apply as in the discussion after Eq. (4). In the LLA, the Reggeized gluon trajectory is needed at 1-loop accuracy and the only contribution to the “real” part of the kernel is from the production of one gluon at Born level in the collision of two Reggeons ($\mathcal{K}_{RRG}^{(B)}$). In the NLA, the Reggeized gluon trajectory is needed at 2-loop accuracy and the “real” part of the kernel takes contributions from one-gluon production at 1-loop level ($\mathcal{K}_{RRG}^{(1)}$) and from two-gluon and $q\bar{q}$ -pair production at Born level ($\mathcal{K}_{RRGG}^{(B)}$ and $\mathcal{K}_{RRQ\bar{Q}}^{(B)}$, respectively). The representation (3) for the elastic amplitude must reproduce with NLA accuracy the representation (1), in the case of exchange of gluon quantum numbers (i.e. color octet representation and negative signature) in the t -channel. This leads to two very stringent “bootstrap” conditions [8]:

$$\frac{g^2 N t}{2 (2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \mathcal{K}^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \omega^{(1)}(t) \omega^{(2)}(t), \quad (8)$$

$$\frac{ig\sqrt{Nt}}{(2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}'^2} \Phi_{A'A}^{(8,a)(1)}(\vec{q}_1, \vec{q}; s_0) = \Gamma_{A'A}^{a(1)} \omega^{(1)}(t) + \frac{\Gamma_{A'A}^{a(B)}}{2} \left[\omega^{(2)}(t) + \left(\omega^{(1)}(t) \right)^2 \ln \left(\frac{s_0}{\vec{q}^2} \right) \right]. \quad (9)$$

The first of them involves the kernel of the generalized non-forward BFKL equation in the octet color representation and at 1-loop order, together with the 1- and 2-loop contributions to the Reggeized gluon trajectory; the second one involves the impact factors in the octet color representation at 1-loop order together with the 1- and 2-loop contributions to the Reggeized gluon trajectory and the PPR effective vertices at Born and 1-loop level. Besides providing a stringent check of the gluon Reggeization in the NLA, the above equations are important since they test, at least in part, the correctness of the year-long calculations which lead to the NLA BFKL equation.

The first bootstrap condition has been considered for the part concerning the quark contribution (massless case) [9]. The ingredients for this equation are the quark contribution to the 2-loop Reggeized gluon trajectory $\omega_Q^{(2)}(t)$ [1] and the quark contribution to the “real” part of the NLA kernel in the octet color representation, $\mathcal{K}_r^{Q(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q})$,

$$\mathcal{K}_r^{Q(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \mathcal{K}_{RRG}^{Q(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) + \mathcal{K}_{RRQ\bar{Q}}^{(8)(B)}(\vec{q}_1, \vec{q}_2; \vec{q}) .$$

$\mathcal{K}_{RRG}^{Q(8)(1)}$ depends on the RRG effective vertex at Born level, $\gamma_{c_1 c_2}^{G(B)}$ [1] and on the real part of the quark contribution to the RRG effective vertex at 1-loop level, $\gamma_{c_1 c_2}^{G(Q)(1)}$ [1]; $\mathcal{K}_{RRQ\bar{Q}}^{(8)(B)}$ depends instead on the $RRQ\bar{Q}$ effective vertex, $\gamma_{c_1 c_2}^{Q\bar{Q}}$ [1]. In Ref. [9] the quark contribution to the “real” part of the kernel of the generalized BFKL equation, $\mathcal{K}_r^{Q(R)(1)}$ has been calculated for singlet and octet color representation in the t -channel and it has been explicitly shown that in the octet case the first bootstrap condition (8) is fulfilled for arbitrary space-time dimension D .

The second bootstrap condition has been considered in the case of quark impact factors [10] and gluon impact factors [11]. In the case of quark impact factors, the intermediate states to be considered in the determination of $\text{Im}\mathcal{A}_{P'P}^{(8,\nu)}$ (see Eq. (4)) are one-quark at 1-loop level and quark-gluon pair at Born level. In the case of gluon impact factors, the intermediate states to be considered are one-gluon at 1-loop level and two-gluon and $q\bar{q}$ -pair at Born level. Let us outline the main points of the calculation in the simpler case of the quark impact factors. The contribution from the one-quark intermediate state involves the convolution of the QQR effective vertex $\Gamma_{QA}^{c(B)}$ at Born level with the QQR effective vertex at 1-loop level $\Gamma_{QA}^{c(1)}$ [12, 13] (here A stands for the colliding quark, Q for the quark in the intermediate state). Differently from $\Gamma_{QA}^{c(B)}$, $\Gamma_{QA}^{c(1)}$ contains also a helicity non-conserving term. The contribution from the quark-gluon intermediate state involves the convolution of two $\{QG\}AR$ effective vertices $\Gamma_{\{QG\}A}^c$ at Born level [13]. Again, this contribution to the quark impact factor will contain both helicity conserving and non-conserving terms. In Ref. [10] the NLA quark impact factors have been given in terms of integrals for arbitrary color state in the t -channel and non-forward case. For the octet color representation it has been explicitly shown that the second bootstrap condition is fulfilled, both for the helicity conserving and non-conserving parts of the quark impact factors and for arbitrary space-time dimension D . It must be stressed that the check of the bootstrap for the helicity conserving part represents a check of correctness of previous calculations more than a real test for the gluon Reggeization, since this part is not quite independent of the calculation of the two-loop correction to the gluon trajectory. The check of the bootstrap for the helicity non-conserving part is instead a really new check of the gluon Reggeization. The explicit calculation of the integrals appearing in the expression for the quark impact factors has been performed in the case of massless quarks [10].

The future work includes the check of the first bootstrap condition for the part concerning the gluon contribution [14]. The ingredients for this equation are the gluon contribution to the 2-loop Reggeized gluon trajectory $\omega_G^{(2)}(t)$ [1] and the gluon contribution to the “real” part of the NLA kernel in the octet color representation, $\mathcal{K}_r^{G(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q})$,

$$\mathcal{K}_r^{G(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \mathcal{K}_{RRG}^{G(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) + \mathcal{K}_{RRGG}^{(8)(B)}(\vec{q}_1, \vec{q}_2; \vec{q}) .$$

$\mathcal{K}_{RRG}^{G(8)(1)}$ depends on the RRG effective vertex at Born level $\gamma_{c_1 c_2}^{G(B)}$ [1] and on the real part of the gluon contribution to the RRG effective vertex at 1-loop level $\gamma_{c_1 c_2}^{G(G)(1)}$ [1], known for arbitrary kinematics only in the $D \rightarrow 4$ limit. The determination of $\gamma_{c_1 c_2}^{G(G)(1)}$ for arbitrary space-time dimension D , in terms of uncalculated integrals (that is enough at least for the check of the bootstrap) is in progress [14]. $\mathcal{K}_{RRGG}^{(8)(B)}$ depends on the RRGG effective vertex $\gamma_{c_1 c_2}^{GG}$ [1] and is known [15, 1]. The final step will be the check that the first bootstrap condition (8) is fulfilled also for the gluon part of the kernel of the generalized non-forward BFKL equation for arbitrary space-time dimension D [14]. This check is already known to be fulfilled in the physical limit $D \rightarrow 4$ [1, 16].

Very recently, another set of “strong” bootstrap conditions has been derived [17, 18], based on the assumption of NLA Reggeization of Reggeon-particle scattering amplitudes. As a consequence of this assumption, it turns out that the impact factors of any colliding particle in the octet color representation are proportional to the related PPR effective vertex, with a universal coefficient function.

Acknowledgments

The author acknowledges the invaluable pleasure coming from the listening of the “morskoi priboi” of the Black Sea during the time of the Conference.

References

- [1] V.S. Fadin, these proceedings; V.S. Fadin, “BFKL News”, preprint Budker INP 98-55, [hep-ph/9807528](#), and references therein.
- [2] M. Gell-Mann *et al.*, Phys. Rev. **133 B** (1964) 145.
- [3] S. Mandelstam, Phys. Rev. **137 B** (1964) 949.
- [4] M.T. Grisaru, H.J. Schnitzer, Phys. Lett. **30** (1973) 811; Phys. Rev. **D 8** (1973) 4498.
- [5] L.N. Lipatov, Yad. Fiz. **23** (1976) 642 [Sov. J. Nucl. Phys. **23** (1976) 338].
- [6] V.S. Fadin, V.E. Sherman, ZhETF Pis'ma **23** (1976) 599; ZhETF **72** (1977) 1640.
- [7] Ya.Ya. Balitskii, L.N. Lipatov and V.S. Fadin, in Proceedings of Leningrad Winter School on Physics of Elementary Particles, Leningrad, 1979, 109.
- [8] V.S. Fadin, R. Fiore, Phys. Lett. **B440** (1998) 359.
- [9] V.S. Fadin, R. Fiore and A. Papa, Phys. Rev. **D60** (1999) 074025.
- [10] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. **D61** (2000) 094006.
- [11] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. **D61** (2000) 094005.
- [12] V.S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. **D50** (1994) 2265.
- [13] V.S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. **D53** (1996) 2729.
- [14] V.S. Fadin, R. Fiore and A. Papa, in preparation.
- [15] V.S. Fadin and D.A. Gorbachev, JETP Lett. **71** (2000) 222; Pis'ma Zh. Eksp. Teor. Fiz. **71** (2000) 322; preprint Budker INP 99-115.
- [16] V.S. Fadin, R. Fiore and M.I. Kotsky, in preparation.
- [17] M. Braun, [hep-ph/9901447](#); M. Braun and G.P. Vacca, Phys. Lett. **B447** (2000) 156.
- [18] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, in preparation.